

Gravitation form-factors and spin asymmetries in hadron elastic scattering

O.V. Selyugin¹

(1) *BLTPh, JINR, Dubna, Russia*

† *E-mail: selugin@theor.jinr.ru*

Abstract

In the framework of the model, where the scattering amplitude is determined by the first and second moments of the GPDs, the qualitative description of all existing experimental data at $\sqrt{s} \geq 52.8$ GeV, including the Coulomb range and large momentum transfers, is obtained with only 4 free parameters. The spin-flip amplitude of the nucleon-nucleon elastic scattering is determined taking into account the spin-dependence part of the second moment of the generalized parton distributions (GPDs) with a new set of t -dependence. The corresponding value of the A_N for the pp at high energy (RHIC) elastic scattering is obtained.

1 Introduction

The dynamics of strong interactions finds its most complete representation in elastic scattering at small angles. Only in this region of interactions can we measure the basic properties that define the hadron structure: the total cross section, the slope of the diffraction peak and the parameter $\rho(s, t)$. Their values are connected on the one hand with the large-scale structure of hadrons and on the other hand with the first principles which lead to the theorems on the behavior of the scattering amplitudes at asymptotic energies [1, 2].

There are many different models for the description of hadron elastic scattering at small angles [3]. They lead to different predictions for the structure of the scattering amplitude at asymptotic energies, where the diffraction processes can display complicated features [4]. This concerns especially the asymptotic unitarity bound connected with the Black Disk Limit (BDL) [5]. saturation regime, which will be reached at the LHC [6, 7]. The effect of saturation will be a change in the t -dependence of B and ρ , which will begin for $\sqrt{s} = 2$ to 6

TeV, and which may drastically change $B(t)$ and $\rho(t)$ at $\sqrt{s} = 14$ TeV [8]. As we are about to see, such a feature can be obtained in very different models.

Spin effect play very often the crushable role for the many different theoretical approaches. Now there are many different models for the description of the elastic hadrons scattering amplitude at small angles [9]. They lead to different predictions of the structure of the scattering amplitude at super-high energies.

The total helicity amplitudes can be written as $\Phi_i(s, t) = \Phi_i^h(s, t) + \Phi_i^{\text{em}}(s, t)e^{\varphi(s, t)}$, where $\Phi_i^h(s, t)$ comes from the strong interactions, $\Phi_i^{\text{em}}(s, t)$ from the electromagnetic interactions and $\varphi(s, t)$ is the interference phase factor between the electromagnetic and strong interactions [10, 11, 12].

In the impact-parameter representation the Born term of the scattering amplitude will be

$$\chi(s, b) \sim \int d^2q e^{i\vec{b}\cdot\vec{q}} F_{\text{Born}}(s, q^2), \quad (1)$$

where $t = -q^2$ and dropping the kinematical factor $1/\sqrt{s(s - 2m_p^2)}$ and a factor s in front of the scattering amplitude. After unitarisation, the scattering amplitude becomes

$$F(s, t) \sim \int e^{i\vec{b}\vec{q}} \Gamma(s, b) d^2b. \quad (2)$$

In this work the standard eikonal unitarisation scheme is used which leads to the standard regime of saturation, *i.e.* the BDL [13]:

$$\Gamma(s, b) = 1 - \exp[-i\chi(s, b)]. \quad (3)$$

The overlap function $\Gamma(s, b)$ can be a matrix, corresponding to the scattering of different spin states. Unitarity of the S -matrix, $SS^\dagger \leq 1$, requires that $\Gamma(s, b) \leq 1$. There can be different unitarization schemes which map $\chi(s, b)$ to different regions of the unitarity circle [14].

In different models one can obtain various pictures of the profile function based on different representations of the hadron structure. In the model [15] we suppose that the elastic hadron scattering amplitude can be divided in two pieces. One is proportional to the electromagnetic form factor. It plays the most important role at small momentum transfer. The other piece is proportional to the matter distribution in the hadron and is dominant at large momentum transfer.

As in the soft-hard pomeron model (EPSH) [8], we take into account the contributions of the soft and hard pomerons. The nucleon-nucleon elastic scattering amplitude is proportional to the electromagnetic hadrons form-factors and can be approximated at small t by

$$T(s, t) = [k_1 (s/s_0)^{\epsilon_1} e^{\alpha'_1 t \ln(s/s_0)} + k_2 (s/s_0)^{\epsilon_2} e^{\alpha'_2 t \ln(s/s_0)}] G_{em}^2(t), \quad (4)$$

where $k_1 = 4.47$ and $k_2 = 0.005$ are the coupling of the “soft” and “hard” pomerons, and $\epsilon_1 = 0.00728$, $\alpha'_1 = 0.3$, and $\epsilon_2 = 0.45$, $\alpha'_2 = 0.10$ are the intercepts and the slopes of the two pomeron trajectories. The normalization s_0 will be dropped below and s contains implicitly the phase factor $\exp(-i\pi/2)$. I shall examine only high-energy nucleon-nucleon scattering with $\sqrt{s} \geq 52.8$ GeV. So, the contributions of reggeons and odderon will be neglected. This model only includes crossing-symmetric scattering amplitudes. The assumption about the hadron form-factors leads to the amplitude

$$T(s, t)_{Born.} = h_1(F_{Born}^s + F_{Born}^h)G_{em}^2 + h_2(F_{Born}^s + F_{Born}^h)G_{grav.}^2, \quad (5)$$

A non-linear trajectory for the pomeron is supposed and, as a first approximation, it is assumed that the coupling is proportional to the gravitational form factor and that both soft and hard terms in the $F_{Born}(s, t)$ have $\alpha' = 0$ at large t .

2 Hadron form factors

As was mentioned above, all the form factors are obtained from the GPDs of the nucleon. The electromagnetic form factors can be represented as first moments of GPDs ([16, 17])

$$F_1^q(t) = \int_0^1 dx \mathcal{H}^q(x, t); \quad F_2^q(t) = \int_0^1 dx \mathcal{E}^q(x, t). \quad (6)$$

Recently, there were many different proposals for the t dependence of GPDs. We introduced a simple form for this t -dependence [18], based on the original Gaussian form corresponding to that of the wave function of the hadron. It satisfies the conditions of non-factorization, introduced by Radyushkin, and the Burkhardt condition on the power of $(1-x)^n$ in the exponential form of the t -dependence. With this simple form we obtained a good description of the proton electromagnetic Sachs form factors. Using the isotopic invariance we obtained

good descriptions of the neutron Sachs form factors without changing any parameters.

The Dirac elastic form factor can be written

$$G^2(t) = h_{fa}e^{d_1 t} + h_{fb}e^{d_2 t} + h_{fc}e^{d_3 t}. \quad (7)$$

with $h_{fa} = 0.55$, $h_{fb} = 0.25$, $h_{fc} = 0.20$, and $d_1 = 5.5$, $d_2 = 4.1$, $d_3 = 1.2$. The exponential form lets us calculate the hadron scattering amplitude in the impact parameter representation [8]. The model used the GPDs of nucleon to obtain the gravitational form factor of the nucleon in the impact-parameter representation. This form factor can be obtained from the second momentum of the GPDs. Taking instead of the electromagnetic current J^μ the energy-momentum tensor $T_{\mu\nu}$ together with a model of quark GPDs, one can obtain the gravitational form factor of fermions

$$\int_{-1}^1 dx x [H(x, \Delta^2, \xi) \pm E(x, \Delta^2, \xi)] = A_q(\Delta^2) \pm B_q(\Delta^2). \quad (8)$$

For $\xi = 0$ one has

$$\int_0^1 dx x [\mathcal{H}(x, t) \pm \mathcal{E}(x, t)] = A_q(t) \pm B_q(t). \quad (9)$$

Calculations in the momentum-transfer representation show that the second moment of the GPDs, corresponding to the gravitational form-factor, can be represented in the dipole form

$$A(t) = L^2/(1 - t/L^2)^2. \quad (10)$$

with the parameter $L^2 = 1.8 \text{ GeV}^2$. For the scattering amplitude, this leads to

$$A(s, b) = \frac{L^5 b^3}{48} K_3(Lb), \quad (11)$$

where $K_3(Lb)$ is the MacDonald function of the third order. To match both parts of the scattering amplitude, the second part is multiplied by a smooth correction function which depends on the impact parameter

$$\psi(b) = (1 + \sqrt{r_1^2 + b^2}/\sqrt{r_2^2 + b^2}). \quad (12)$$

The model has only four free parameters, which are obtained from a fit to the experimental data:

$$h_1 = 1.09 \pm 0.004; \quad h_2 = 1.57 \pm 0.006; \quad r_1^2 = 1.57 \pm 0.02 \text{ GeV}^{-2}; \quad r_2^2 = 5.56 \pm 0.06 \text{ GeV}^{-2}.$$

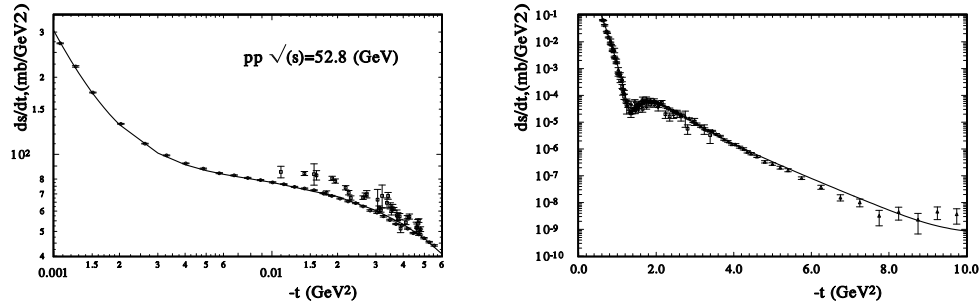


Figure 1: $d\sigma/dt$ at $\sqrt{s} = 52.8$ GeV for pp elastic scattering, at small $|t|$ (left) and at large $|t|$ (right).

It was used all the existing experimental data at $\sqrt{s} \geq 52.8$ GeV, including the whole Coulomb region and up to the largest momentum transfers experimentally accessible. In the fitting procedure, only statistical errors were taken into account. The systematic errors were used as an additional common normalization of the experimental data from a given experiment. As a result, one obtains $\sum \chi_i^2/N \simeq 3$ where $N = 924$ is the number of experimental points [?]. If one sums the systematic and statistical errors, the χ^2/N decreases, to 2. Note that the parameters are energy-independent. The energy dependence of the scattering amplitude is determined only by the intercepts of the soft and hard pomerons. In Fig. 1 the differential cross sections for proton-proton elastic scattering at $\sqrt{s} = 52.8$ GeV are presented. At this energy there are experimental data at small (start at $-t = 0.0004$ GeV²) and large (up to $-t = 10$ GeV²) momentum transfers. The model reproduces both regions and provides a qualitative description of the dip region at $-t \approx 1.4$ GeV², for $\sqrt{s} = 53$ GeV² and for $\sqrt{s} = 62.1$ GeV².

Let us examine the $p\bar{p}$ differential cross sections. In this case at low momentum transfer the Coulomb-hadron interference term plays an important role and has the opposite sign. The model describes the experimental data well. In this case, the first part of the scattering amplitude determines the differential cross sections, and is dominated by the exchange of the soft pomeron. The high energy data at $\sqrt{s} = 630$ and 1800 GeV also describe sufficiently well. There was a significant difference between the experimental measurement of ρ , the ratio of

the real part to the imaginary part of the scattering amplitude, between the UA4 and UA4/2 collaborations at $\sqrt{s} = 541$ GeV. A more careful analysis [19, 20] shows that there is no contradiction between the measurements of UA4 and UA4/2. Now the present model gives for this energy $\rho(\sqrt{s} = 541\text{GeV}, t = 0) = 0.163$, so, practically the same as in the previous phenomenological analysis.

Saturation of the profile function will surely control the behavior of σ_{tot} at higher energies and will result in a significant decrease of the LHC cross section. For the last LHC energy $\sqrt{s} = 14$ TeV the model predicts $\sigma_{tot} = 146$ mb and $\rho(0) = 0.235$. This result comes from the contribution of the hard pomeron and from the strong saturation from the black disk limit.

3 Analysing power at high energies

There is a large spin program at RHIC. This program includes measurements of the spin correlation parameters in the diffraction range of elastic proton-proton scattering. The differential cross section and analyzing power A_N are defined as follows:

$$\frac{d\sigma}{dt} = \frac{2\pi}{s^2}(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2), \quad (13)$$

$$A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} \text{Im}[(\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4)\Phi_5^*], \quad (14)$$

in terms of the usual helicity amplitudes Φ_i .

In the general case, the form of A_N and the position of its maximum depends on the parameters of the elastic scattering amplitude: σ_{tot} , $\rho(s, t)$, the Coulomb-nuclear interference phase $\varphi_{cn}(s, t)$ and the elastic slope $B(s, t)$. For the definition of new effects at small angles, and especially in the region of the diffraction minimum, one must know the effects of the Coulomb-nuclear interference with sufficiently high accuracy. The Coulomb-nuclear phase was calculated in the entire diffraction domain taking into account the form factors of the nucleons [10, 11, 12].

The total helicity amplitudes can be written as

$$\Phi_i(s, t) = \phi_i^h(s, t) + \phi_i^{\text{em}}(t) \exp[i\alpha_{\text{em}}\varphi_{cn}(s, t)]. \quad (15)$$

In this paper, we define the hadronic and electromagnetic spin-non-flip amplitudes as

$$F_{\text{nf}}^h(s, t) = [\phi_1^h(s, t) + \phi_3^h(s, t)]/2s; \quad F_{\text{nf}}^c(s, t) = [\phi_1^{\text{em}}(s, t) + \phi_3^{\text{em}}(s, t)]/2s \quad (16)$$

and spin-flip amplitudes as

$$F_{\text{sf}}^h(s, t) = \phi_5^h(s, t)/s; \quad F_{\text{sf}}^c(s, t) = \phi_5^{\text{em}}(s, t)/s. \quad (17)$$

Equation (14) was applied at high energy and at small momentum transfer, with the following usual assumptions for hadron spin-flip amplitudes: $\phi_1 = \phi_3$, $\phi_2 = \phi_4 = 0$; the slopes of the hadronic spin-flip and spin-non-flip amplitudes are equal.

According to the standard opinion, the hadron spin-flip amplitude is connected with the quark exchange between the scattering hadrons, and at large energy and small angles it can be neglected. Some models, which take into account the non-perturbative effects, lead to the non-dying hadron spin-flip amplitude [21]. Another complicated question is related with the difference in phases of the spin-non-flip and spin-flip amplitude.

Let us suppose that at high energies the spin-flip amplitude will be proportional the first and second momentum of the spin-depended part of the GPDs.

$$F_{\text{sf}}^h(s, t) = i\beta \left(\int \mathcal{E}^q(x, t) dx + \int x \mathcal{E}^q(x, t) dx \right). \quad (18)$$

Here we take only Born term of the spin flip amplitude, as it is small relative spin-non-flip amplitude and examine $A_N(s, t)$ only at non-small t . The coefficient β is unknowing constant, which size $\beta = -0.01\sqrt{|t|}/m_p$ is determined by the size of the diffraction minimum at the $\sqrt{s} = 52.8$ GeV. We do not know the relative sizes of the imaginary and real part of the spin-flip amplitude. For the simplicity in our model we take it as pure imaginary and without energy dependence. So, the ratio of the spin-flip to the spin-non-flip amplitude will be decreasing as $\ln^2(s)$. Our calculations of the $A_N(s, t)$ was shown at Fig.2 and Fig.3. In Fig.2 the calculation draw to low energy $\sqrt{s} = 23.4$ GeV to compare with the existence the experimental data. Our model is essentially high energy approximation - it do not taking into account the contribution of the masses regions. However, the existence of the experimental data do not show the contradiction with our calculations. In fig.3 the predictions our model for the energies of RIHC are shown. The size of the A_N for the maximal energy is not small and really can be measured.

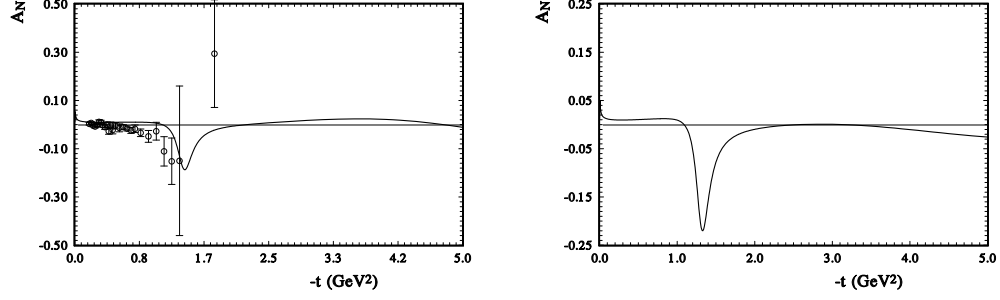


Figure 2: A_N at $\sqrt{s} = 23.4$ GeV [left panel] and at $\sqrt{s} = 50$ GeV [right panel]; experimental points from at $\sqrt{s} = 23.4$ GeV.

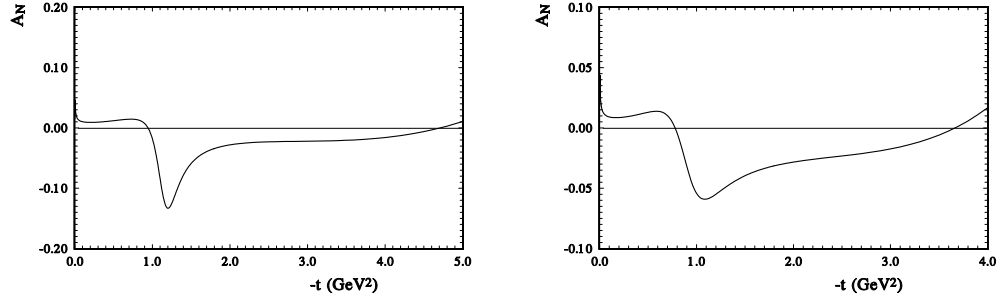


Figure 3: A_N at $\sqrt{s} = 200$ GeV [left panel] and at $\sqrt{s} = 500$ GeV [right panel] calculated in the framework of the model.

4 Conclusion

A new model, taking into account the contributions of the soft and hard pomerons and using form factors calculated from the GPDs, successfully describes all the existing experimental data on elastic proton-proton and proton-antiproton scattering at $\sqrt{s} \geq 52.8$ GeV, including the CNI region, the dip region, and the large-momentum-transfer region. The behavior of the differential cross section at small t is determined by the electromagnetic form factors, and at large t by the matter distribution (calculated in the model from the second momentum of the GPDs). The spin flip amplitude which is determined by the spin-depended parts of the first and second momentum of the GPDs is calculated into the frame work of the model. It is need note that at large momentum transfer the spin-flip amplitude is determined in most part by the gravimagnetic second part of the GPDs $B(t)$. hence the measure of the analizing power at large momentum transfer and large energies can be help to determining its the parameters. The model is super simplified. Especially it is connect to the form of electromagnetic form factor, which is taken as the three exponents to calculate it's form by analitic in the impact parameter representation. The hard form factor also taking into account in the simple dipole form. The slope of the hard part of the Regge form of Pomeron taken as zero. It is means that really slope require the non-linear dependence of momentum transfer. It is need further develop of the model without changes the basis which gives the new view point on the hadron interactions at high energies.

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